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# A new Fractal Series Expansion based enhancement model for license plate recognition



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# ABSTRACT

License plate recognition is an emerging topic for real-time applications in smart city development because of automatic systems for toll fee-paying, traffic controlling, and vehicle detection. Speed of vehicles, unpredictable weather conditions such as night/low light/limited light images, and capturing images at different angles make recognition harder. This paper presents a new Fractal Series Expansion (FSE) model for license plate image enhancement. The proposed FSE model is justified because it estimates the high probability for the pixels which represent the license plate compared to the pixels which represent background irrespective of the above challenges, resulting in enhanced image. Besides, since the FSE model considers local information for estimating probability, the model has the ability to tackle non-uniform degradations as well as distortion affected multiple adverse factors. In addition to qualitative results, to validate the effectiveness of the proposed enhancement, quantitatively, recognition rates of the different methods before and after enhancement are computed. For this purpose, we have considered different datasets like dataset of Night License Plate Images (NLPI), which consists of images captured in the night and low lights environment, the UCSD benchmark dataset which provides poor and high quality day license plate images, etc. It is noted that recognition results after enhancement is higher than that of before enhancement, and hence our enhancement is useful and effective.

# 1. Introduction

Smart city development is the main target of most developing and developed countries such as Malaysia, India, China, and other countries. As a result, developing an automatic system for paying toll fees and parking irrespective of time and weather is still an open issue for the researchers in signal and image processing [1,2]. In addition, recognizing illegal parking in towns automatically and optimal car parking make the problem more interesting and demands immediate solutions. If we capture images in such an environment where one can expect unpredictable weather conditions, such as low light/limited light, it affects quality of images. This is the main threat to the methods developed in the past because most of them address challenges caused by images captured in the day but not night and poor light environment. For instance, Li et al. [3] proposed end-to-end car license plate detection and recognition with deep neural networks, which focuses on challenges of complex background and effects of nonuniform illumination. Panahi et al. [4] proposed accurate detection and

recognition of dirty vehicle plate numbers for high-speed applications, which aims at addressing the problem of noises and blurred images. Hendry and Chen [5] proposed automatic license plate recognition via sliding window darknet-YOLO deep learning, which targets developing an efficient algorithm for particular types of license plate recognition. Khare et al. [6] proposed a novel character segmentation–reconstruction approach for license plate recognition, which finds a solution to images affected by multiple adverse factors.

Similarly, methods are developed in the past for recognizing texts in natural scene images. The methods focus on addressing issues of low contrast, low resolution, complex background, arbitrarily orientation, font size or font variations, multi-lingual texts and some extent to distortion, such as perspective. For example, Luo et al. [7] explore a multi-object rectified attention network for scene text recognition. The method aims at solving issues such as poor quality and distortion affected by perspective distortion. Shi et al. [8] proposed an end-toend trainable neural network for image-based sequence recognition and

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Received 17 September 2019; Received in revised form 3 July 2020; Accepted 2 August 2020 Available online 6 August 2020 0923-5965/© 2020 Elsevier B.V. All rights reserved. its application to scene text recognition. Unlike the existing methods that ignore the importance of the sequence of pixels for improving recognition performance, the method proposes an approach for choosing an optimal sequence such that recognition performance improves greatly with the minimum computational expenses. As a result, it can handle arbitrarily orientations of texts. However, the method may not work well for multi-lingual texts due to image sequence constrain. To overcome this problem, Busta et al. [9] proposed an unconstrained endto-end method for multi-lingual scene text recognition. The method employs fully convolutional neural networks along with Connectionist Temporal Classification for recognizing texts in natural scene images. The method has been trained for different languages for recognizing multi-lingual texts. However, the performance of the method is not good for texts of arbitrarily shaped characters. This is due to different architectures for handling multi-lingual texts. Although the methods address a few issues which are the same as license plate recognition, they are not tested on images captured in the night or low light conditions. These images are affected by not only poor quality but also illumination effect. Therefore, the above natural scene text recognition methods may not perform well for the proposed license plate images.

It is noted from the above discussions that although the methods attempted to solve all the possible challenges of license plate images, none of them considered images captured in the night/low light/limited light environments for license plate recognition. The same conclusion can be drawn from recognition methods of natural scene text. This shows that the methods may not perform well for poor quality images due to light condition. This observation motivates us to develop a novel model for license plate image enhancement such that we can expand the scope of the above methods for improving recognition performances. It is evident from the illustration shown in Fig. 1, where the recognition results shown in a double quote for the input images of different datasets in Fig. 1(a) are erroneous. On the other hand, recognition results reported for enhanced images obtained by the proposed model (shown in Fig. 1(b)) are better compared to those in Fig. 1(a) for all the sample images of different datasets. For recognizing license plates in the images in Fig. 1, we consider the recognition method [7] that uses a multi-object rectified attention network for scene text recognition. The reason to consider this method for recognition experiments is that it involves a strong powerful different deep learning model for addressing challenges, which are similar to the proposed license plate recognition. Besides, texts in license plate images are the same with those in natural scene images. Therefore, one can conclude from the results shown in Fig. 1 that the existing recognition method may not be adequate for addressing challenges posed by night/low light/limited light license plate images.

#### 2. Related work

It is noted from the methods developed for text detection in natural scene and license plate images that the methods are robust to low or poor quality and some extent to distortion caused by blur and perspective distortion. As a result, the performance of text detection may not degrade much for images captured in night or low light conditions. Shivakumara et al. [10] proposed a Fractional means based method for multi-oriented keyword spotting in video/scene/license plate images. This method addressed challenges of images affected by blur, noise, illumination effect, low contrast and complex background. Asif et al. [11] proposed multinational vehicle license plate detection in complex backgrounds, which addresses the issues of illumination effect and different environmental conditions. Shemarry et al. [12] proposed an ensemble of adaboost cascades of 3L-LBPs classifiers for license plate detection in low quality images. The method focuses on the challenges of outdoor environments such as sun light, complex background and weather conditions. Panahi et al. [4] proposed accurate detection and recognition of dirty vehicle plate numbers for high-speed applications. The method works well for images affected by blur and noises. It is noted from the above methods that the challenges do not affect for detection performance. Therefore, this work considers the output of license plate detection methods as the input for enhancement. Hence, this section reviews the papers related to general image/license plate image enhancement.

Liu et al. [13] proposed a contrast enhancement model based on dark channel prior. The method aims at removing haze from images based on the combination of illumination and color information. However, the method is good for images affected by haze but not poor quality caused by night vision and other degradations. Noor et al. [14] proposed multi-scale gradient image super-resolution for preserving SIFT key points in low resolution images. The method targets low resolution images of surveillance applications. To achieve this goal, the method proposed combination of gradient and deep learning models. This method requires low resolution images for enhancing. In our case, captured images may not have uniform resolution and contrast. Therefore, it may not perform well for our work. Shan et al. [15] proposed robust contrast enhancement of forensic documents based on convolutional neural networks. The method focuses on images affected by forgery operation. For the purpose of enhancing the distortion introduced during forgery operation, the method explores the combination of CNN and gray-level co-occurrences matrix. Since the method is developed for particular distortion, it may not work well for night/low light and limited light images. Jammal et al. [16] proposed multi-view video quality enhancement without depth information. The method considers poor quality and degradations caused by compression for image enhancement. The method uses deep learning models for enhancing the quality of images. This method is good for video with temporal information but not still images. Tohl and Li [17] proposed contrast enhancement by multi-level histogram shape segmentation with adaptive detail enhancement for noise suppression. The method finds edge information for enhancing low contrast information in images. Histogram operation helps the method to find abrupt changes in images, which suppress homogeneous regions, resulting in edges. This method is good when edges provide high contrast information. However, when there is a loss of edges, the method underperforms. Loh et al. [18] proposed low-light image enhancement using Gaussian process for feature retrieval. The method addresses the issue of the proposed work by exploring a convolutional neural network. The method considers low light enhancement is collection Gaussian functions. Then the same is trained by convolutional neural networks. The main issue of the method is that how to choose the number of Gaussian functions for different low light images. Shamasneh et al. [19] proposed a new local fractional entropy based model for Kidney image enhancement. Since the objective of the method is to enhance particular kidney regions, the method is tuned and localized based on the knowledge of kidney information. Therefore, the method is limited to kidney images.

In summary, the methods are proposed for addressing several challenges of general image enhancement. It is noted from the above review that none of the methods focuses on degraded texts in images. In addition, most of the methods targets particular causes or distortions for finding a solution. Therefore, we can assert that the methods are good for enhancing the whole image (global enhancement) but not for local enhancement such as texts or license plates in images.

To alleviate the above limitation, methods have been developed for enhancing text information in degraded or poor quality images. For example, Roy et al. [20] proposed a Fractional Poisson enhancement model for text detection and recognition in video frames. Though the method aims at enhancing fine details of texts, it is not for images affected by low light or night vision but rather images affected by Laplacian operation. Therefore, the method is limited to noise images. Rabbani et al. [21] proposed a method for Bangladeshi license plate image enhancement. The method follows the conventional way, namely, binarization and thresholding, for enhancing license plate images. This idea may work well for Bangladeshi license plate images but not necessarily for other license plate images because binarization

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(b) Proposed enhancement results for corresponding images in (a) and their recognition results for enhancement

"lvik395"

"frequent"

Fig. 1. Illustrating the performance of the recognition method [7] before and after enhancement.

and thresholding are not effective for complex background images. Min et al. [22] proposed a new approach for license plate image enhancement based on YOLO-L model. The method focuses on challenges caused by weather conditions by exploring the deep learning model. However, the scope of the method is confined to weather conditions but not light conditions. Shemarry et al. [23] proposed an efficient texture descriptor for the detection of license plates from vehicle images in difficult conditions. In this method, Gaussian filters and adaptive histogram equalization enhancement have been used for license plate image enhancement. This idea is good for distortions caused by different situations such as low contrast, complex background and blur. However, it is not clear how the method works for images with loss of visibility and variations on quality. Raghunandan et al. [24] proposed a Riesz Fractional based model for enhancing license plate detection and recognition. In this method, Fractional calculus has been used for enhancing license plates in degraded images. The method considers degradations caused by low contrast, low resolution, blur and complex background but not for low light/limited light and night images, where one can expect unpredictable quality and loss of visibility.

"please"

In summary, there are methods proposed in the literature for addressing issues of poor quality license plate images and texts in images. However, the scope of the methods is limited to poor quality caused by different degradations but not for low light, limited light, or night environments. Therefore, the above methods may not be effective for license plates affected by the above-mentioned light conditions. This is due to low light/limited light/night, which may cause not only poor quality but also loss of visibility, contrast and non-uniform light illumination effect. Hence, enhancing such license plate images is much more challenging and complex. Recently, there is an attempt to address challenges of low light images by Zhang et al. [25], who propose new fusion based enhancement for text detection in night video footage. The method combines the advantage of spatial and frequency information for fusing images. However, the performance of the method is not satisfactory because it is not sure when spatial or frequency information provides fine details of texts. In other words, the performance of the method depends on the success of both the steps. Therefore, the method is not effective for different situations. To overcome the problem of the method [25], Pinaki et al. [26] proposed a method based on deep learning models. The method explores U-net based architecture for enhancing license plates of day and night images. Since the Unet architecture extracts context and symmetry information of text, the method works well for both day and night images. However, the performance of the method heavily depends on learning, samples and training. Thus, the method may not work well for different situations, such as images of different scales, and images affected by different degrees of distortion.

Hence, in this work, we propose a new model based on Fractal Series expansion for license plate image enhancement. It is true that when an image is affected by low light, limited light and night conditions, one cannot expect the uniform degradations/changes in it, which results in uncertainty of choosing correct pixels as informative. In other words, the effect of degradation is non-linear. Motivated by the special property of Fractals which are good for finding a solution to non-linear problems [20,24], we explore Fractal Series Expansion (FSE), which estimates the probability through energy of pixels being classified as fine edges based on local information for handling the uncertainty of edge pixels. It is true that edge information is prominent, which represents texts with high energy compared to its background pixels in license plate images. Therefore, the objective of the proposed model is to enhance such edges irrespective of degree of poor quality caused by low light/limited light and nigh light conditions.

"a4"

Thus the contribution of the proposed work is as follows. (1) Introducing Fractional Series Expansion (FSE) for addressing challenges of license plate enhancement such that the recognition performance improves, and (2) creating a new dataset called Night License Plate Images (NLPI), which provides images captured in low/limited light environment and different weather conditions for experimentation. At the same time, in order to support Research Reproducibility, we plan to release the dataset to the public upon acceptance.

### 3. Proposed model

In this work, we consider license plate regions detected by the method [10] for image enhancement. This is because the poor quality or distortion caused by low/limited/night light affects greatly for recognition but not detection. In other words, to achieve better recognition, structures of characters in license plates need to be preserved. In addition, especially for license plate recognition, real time applications expect high and consistent results without any errors or mistakes. On the other hand, in case of detection, if the detection method misses a few pixels or characters, it is possible to restore the whole license plate area.

As mentioned in the previous section, for enhancing fine details of license plate images, we explore Fractal Series Expansion (FSE) in this work, which comprises two sub-sections, namely, an overview of FSE and description of the recognition methods [7-9]. However, for recognition, we choose the state-of-the-art method that uses powerful deep learning models. The reason for choosing the existing recognition methods is that though these methods are developed for text recognition in natural scene images, they are robust and work well for almost all the situations irrespective of text types. The proposed work consists of two sub-sections, namely, sub-section 1 that presents a new enhancement model, and sub-section 2 that describes the above-mentioned threerecognition methods [7-9] for validating the proposed enhancement model. In the proposed enhancement model, for the input image, it estimates the probability of pixels that represent license plates, fractional dimension followed by FSE calculation. Further, the results of FSE are convolved with the input image, resulting in an enhanced

image. The enhanced image is then fed to the recognition method to validate enhanced results. The block diagram of the proposed work is shown in Fig. 2.

### 3.1. Overview of fractal series expansion

Fractal series expansion is one of the important methods of fractal analysis used to measure fractal features of data. It involves a formal approach to assign a fractal dimension ( $\alpha$ ) and other fractal features (e.g., probability of pixels, entropy of pixels) to a dataset, which can be signals, or images. This type of analysis is now extensively used for solving complex issues in the field of signal and image processing. We explore the same to formulate a new fractal series entropy. The model is implemented through energy estimation by Fractional Tsallis Entropy (FTE), which works based on the probability value of each pixel as well as its marginal FTE. The formulation of FSE is as follows.

Fractal series expansion of two functions is given in the following form [26,27].

$$(f+g)^{n\alpha} = \sum_{k=0}^{n} \binom{n\alpha}{k\alpha} f^{k\alpha} g^{(n-k)\alpha}$$
(1)

where  $\binom{n\alpha}{k\alpha} = \frac{\Gamma(1+n\alpha)}{\Gamma(1+k\alpha)\Gamma(1+(n-k)\alpha)}$ ,  $\Gamma$  is an Euler gamma function, and  $\alpha$  is the fractional power operator.

Inspired by the method [27–29] where it is stated that Tsallis nonextensive entropy outperforms other entropy formulation, we explore the same for entropy calculation using pixels in license plate image. The mathematical steps of Tsallis non-extensive entropy are as follows. Since an image has discrete gray levels of pixels (*i*), therefore the probability of a pixel in the input image is denoted by  $\rho$ i, with random variable *i*.

The Tsallis entropy of order  $0 < \alpha < 1$  is defined as:

$$T_{\alpha}(\rho(u)) = \frac{\int_{a}^{b} (\rho(u))^{\alpha} du - 1}{1 - \alpha}.$$
 (2)

Hence, in the discrete form, we have:

$$T_{\alpha}(\rho) = \frac{1}{1-\alpha} \left( \sum_{k=0}^{n-1} \rho^{\alpha}{}_{k} - 1 \right).$$
(3)

By considering the entropy of one pixel from (3), we obtain

$$T_{\alpha}\left(\rho_{i}\right) = \frac{1}{1-\alpha}(\rho_{i}^{\alpha}-1)$$
(4)

Note that  $\rho_i < 1$  is the probability of the pixel *i*.

By taking the derivative of (4) with respect to  $\rho_i$ , we have the differential entropy

$$\mathbf{T}'_{\alpha}\left(\rho_{i}\right) = \frac{\alpha}{1-\alpha}\left(\rho_{i}^{\alpha-1}\right).$$
(5)

Combining the power function  $\rho_i^{\alpha}$  (refers to *f* in (1)) and its fractional entropy in (5) (refers to *g* in (1)) in the fractal series (1), we have the fractal series entropy formula as

$$\left(\rho_{i}+T'_{\alpha}(\rho_{i})\right)^{n\alpha}=\sum_{k=0}^{n}\binom{n\alpha}{k\alpha}\rho_{i}^{k\alpha}\left(T'_{\alpha}\left(\rho_{i}\right)\right)^{(n-k)\alpha}$$
(6)

Thus, (6) represents the local fractional difference derivative operator of the probability  $\rho_i$  based on the derivative of Tsallis entropy.

Eq. (6) can be viewed by the following form

$$D^{\alpha}_{i} = \sum_{k=0}^{n} \frac{\Gamma(1+n\alpha)}{\Gamma(1+k\alpha)\Gamma(1+(n-k)\alpha)} \rho_{i}^{k\alpha} \left(T'_{\alpha}\left(\rho_{i}\right)\right)^{(n-k)\alpha}, \quad \rho_{i} \neq 0,$$

$$= \sum_{k=0}^{n} \frac{\Gamma(1+n\alpha)}{\Gamma(1+k\alpha)\Gamma(1+(n-k)\alpha)} \rho_{i}^{k\alpha} \left(\frac{\alpha}{1-\alpha}\left(\rho_{i}^{\alpha-1}\right)\right)^{(n-k)\alpha}$$

$$= \sum_{k=0}^{n} \frac{\Gamma(1+n\alpha)}{\Gamma(1+k\alpha)\Gamma(1+(n-k)\alpha)} \left(\frac{\alpha}{1-\alpha}\right)^{(n-k)\alpha} \rho_{i}^{k\alpha} (\rho_{i}^{\alpha-1})^{(n-k)\alpha}$$

$$= \sum_{k=0}^{n} \frac{\Gamma(1+n\alpha)}{\Gamma(1+k\alpha)\Gamma(1+(n-k)\alpha)} \left(\frac{\alpha}{1-\alpha}\right)^{(n-k)\alpha} \rho_{i}^{\alpha[k+(\alpha-1)(n-k)]}$$
(7)

where *n* indicates the number of pixels, and  $D^{\alpha}_{\ i} = (\rho_i + T'_{\alpha}(\rho_i))$ .

From (7), by approximating the sum when n-k = 1 and minimizing it, we have the coefficient formula  $\omega_i \approx \frac{\Gamma(1+(1+k)\alpha)}{\Gamma(1+k\alpha)\Gamma(1+\alpha)} \left(\frac{\alpha}{1-\alpha}\right)^{\alpha} \rho_i^{\alpha[k+(\alpha-1)]}$ ,  $i = 1, 2, ..., < \alpha < 1$ , for fixed k = 1 (first iteration), we obtain

$$\Omega \approx \frac{\Gamma(1+2\alpha)}{(\Gamma(1+\alpha))^2} \left(\frac{\alpha}{1-\alpha}\right)^{\alpha} \left(\rho_i\right)^{\alpha^2}, \qquad 0 < \alpha < 1,$$
(8)

By using the local fractional quantity ( $\Omega$ ), an enhanced image  $I_{\rm F}$  is formulated by:

$$I_F = \frac{\Gamma(1+2\alpha)}{(\Gamma(1+\alpha))^2} \left(\frac{\alpha}{1-\alpha}\right)^{\alpha} (\rho_i)^{\alpha^2} . I$$
(9)

where *I* is the input image. The fractional power values ( $\alpha$ ) is given by the range of  $0 < \alpha \le 1$ .

For each pixel in the input image, the proposed model derives local FSE, which results in a probability matrix.

The effect of FSE is shown in Fig. 3, where we can for the images shown in Fig. 3(a), the statistical means and the standard deviations report high values for after enhancement, while low values for before enhancement as shown in Fig. 3(b). This shows that the proposed FSE model suppresses homogeneous information and enhances abrupt changes (edges) in images. It is valid because when there are high values with large variations due to abrupt changes, the mean and standard deviation should be high. It is evident from the output of the proposed FSE model shown in Fig. 3(a), where the pixels that represent background are suppressed, while the pixels that represent license plates are highlighted.

#### 3.2. License plate recognition

As discussed in the Proposed Model Section, we choose three recognition methods of different models for validating the output of the proposed enhancement model. Shi et al. [8] proposed an end-to-end trainable neural network for image-based sequence recognition and its application to scene text recognition. Busta et al. [9] proposed an unconstrained end-to-end method for multi-lingual scene text recognition. The method uses fully convolutional neural networks along with Connectionist Temporal Classification for recognizing texts in natural scene images. Luo et al. [7] proposed a multi-object rectified attention network for scene text recognition. The proposed architecture in this method is generalized and works for general scene text recognition, which includes arbitrarily shaped texts. To achieve this, the method employs a multi-object rectified network and an attentionbased sequence recognition network. To address noise perturbations, the method expands the visual field, which further improves sensitivity of the attention based decoder. Overall, the above recognition methods are capable of handling license plate recognition after enhancement. The network architecture of the recognition methods can be seen in Fig. 4(a) and (b).

Both Shi et al. [8] and Busta et al. [9] essentially follow a CRNN framework for text recognition as shown in Fig. 4(a). The CRNN framework considers cropped word images and passes through a series of convolutional layers and pooling layers, which results in a feature map of shape 1×W×D, where 'W' represents width, while 'D' represents depth of the feature map. Note that the height is equal to 1. The results of this step are considered as the input for a recurrent neural network, with each "1×W×D" being fed as the input at each time step. The recurrent neural network encodes this information and outputs a 2 dimensional vector, for which CTC (Connectionist Temporal Classification) loss is calculated for training. For testing, the CTC decoding algorithm is applied on the 2 dimensional vector to retrieve a predicted string. Luo et Al.'s method [7] consists of two key steps for prediction, namely, a multi-object rectification network, and a recognition network using a RNN cell for decoding with attention instead of CTC as shown in Fig. 4(b). The rectification network is useful to tackle challenges caused by various shapes and irregular patterns. The decoding is done using a











#### (b) Mean and standard deviation of Fractional matrix of the images of before and after enhancement

#### Fig. 3. Illustration of the proposed enhancement models for input and output images.



(b) The Network architecture of the recognition method [7]

Fig. 4. Architectures of the chosen recognition methods.

Gated Recurrent Unit (GRU) along with attention to the encoder BLSTM (Bidirectional LSTM).

To train the above-recognition models, we consider ICDAR 2017 MLT cropped word images. Then we perform recognition experiments for input images. The proposed work trains the models with enhanced images given by the proposed enhancement model of the ICDAR 2017 MLT dataset. This model is used for recognition experiments on enhanced images of different datasets. More details about the learning can be found in [7–9].

#### 4. Experimental results

To evaluate the proposed enhancement model, we create our own dataset, which is called the Night License Plate Image (NLPI) dataset

#### Table 1

etails of the different databases includin	g our license plate data	aset and natural scene datasets.
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Databases	Testing	Resolution of license	Blur	Script	Imbalanced	Orientation	Perspective
	samples	plate regions		MLT	Illumination		
NLPI (Our dataset)	200	Min: 31×12		>		<b>~</b>	
		Max: 504×1358					•
Day-UCSD-1	1290	Min: 16×19					
		Max: 81×122					
Day-UCSD-2	114	Min: 11×31		>			
		Max: 26×81					
Day-UCSD-3	291	Min: 12×25				*	
		Max: 87×179					



UCSD-3

NLPI (Our dataset)

Fig. 5. Some samples of license plate images of benchmark and our datasets.

because there is no standard dataset available in the literature. We use the text detection method in [10] for extracting license plate regions from the captured images. The dataset includes images captured at different timings from evening to night, imbalanced illumination images, different orientated text images, and images affected by perspective distortions as shown sample images in Fig. 5 (night dataset). To test the objectiveness of the proposed model, we consider detected license plate images of the standard datasets, which provide only day images, namely, UCSD-1, UCSD-2 and UCSD-3 [6]. UCSD-1 consists of images with blur and low-resolution images as the shown sample images in Fig. 5. UCSD-2 consists of images of the severe blur with noises as shown in Fig. 5. UCSD-3 comprises images with good focus as the shown images in Fig. 5. Statistical details including characteristics and numbers of testing samples for all the datasets are listed in Table 1, where different attributes of datasets can be seen. Table 1 shows that the considered datasets for experimentation and evaluation have large variations in terms of resolution and artifacts like blur, perspective distortion, orientation, script and illumination. In total, 1895 images are considered for evaluating the proposed and existing methods. Therefore, we believe that the considered datasets provide possible variations for evaluating robustness, effectiveness, and usefulness of the proposed method.

For measuring the performance of the proposed enhancement model, we consider NIQE (Naturalness Image Quality Evaluator) which measures image quality, and PIQE (Perception based Image Quality Evaluator) which measures visual quality because these measures do not require the ground truth and reference images [30,31]. The datasets considered in this work do not have ground truth; therefore, these measures are suitable for evaluating the quality of the enhanced images. In addition, NIQE and PIQE are no reference image quality scores, which evaluate both image quality as well perceptual (visual) quality. A low score of NIQE or PIQE indicates an enhanced image has better quality. To show the effectiveness of the proposed enhanced model, we calculate the above two measures before enhancement (input image) and after enhancement (enhanced images). It is expected both the measure should score low for the enhanced images compared to before enhancement. At the same time, to validate the enhanced results, the proposed method calculates recognition rate using distance measures. The distance measures are calculated using labeled samples, and the results are given by the methods. Recognition rate is defined

as the number of characters recognized correctly divided by the actual number of characters.

To show the usefulness of the proposed enhancement model, the proposed enhanced results are compared with classical methods, namely, Histogram Equalization (HE), Contrast Limited Adaptive Histogram Equalization (CLAHE), and Adjust Intensity Values to Specified Range (AIV). The reason to consider the above classical methods for comparative study is to show that the baseline methods are not capable in handling license plate images captured in low light and night conditions. Apart from the above classical methods, we also implement four enhancement methods, namely, the method in [21] which works based on local fractional entropy, the method in [24] which works based on Riesz Fractional calculus, the method in [25] which works based on the combination of spatial and frequency domain for low contrast license plate enhancement, and the method in [26] which works based on a deep learning model for enhancing low light and day license plate images for comparative studies with the proposed model. The reason to consider the method in [21] is that it is developed for general image enhancement. Similarly, the method in [24] aims at enhancing low quality license plate images as the proposed method. Besides, both the methods [21,24] use the concept of Fractional calculus as the proposed method. The methods [25,26] are developed for enhancing low contrast license plate images, which is the same objective of the proposed method. To show that the methods [21,24-26] are not effective for images captured in low light environment, we consider them for comparative studies.

Similarly, to validate the enhanced results given by enhancement methods, we run the text recognition methods in [7–9] presented in Section 3.2 for calculating recognition rates for the output of the proposed and existing enhancement models. The motivation to choose these three methods is discussed in the same Section. It is expected that the recognition performances of the methods should be better for enhanced images compared to input images in all the datasets.

In the proposed enhancement model, fractal dimension is the key parameter for enhancing fine details in images of low light conditions. To determine the optimal value for the parameter ( $\alpha$ ), we calculate the average PIQE for different values of alpha on our night images as shown in Fig. 6, where it is observed that at 0.75, PIQE gives the lowest compared to other values of alpha. Therefore, we consider 0.75 as the value of alpha for all the experiments in this work.



Fig. 6. Experiment to decide the value of alpha based on the average PIQE measure.

#### 4.1. Evaluating the proposed enhancement model

Qualitative results of the proposed and existing models on our night dataset and benchmark datasets are shown in Fig. 7, where it is noted that the proposed model is better than the existing models for almost all the four datasets. This shows that the proposed model can be used for enhancing poor quality of day images as well as images captured in low light conditions. The reason for this success is that the probability of a pixel along with entropy makes difference for both day and night images as it is insensitive to day and night images. Therefore, the proposed model is good for both the poor qualities of night and day images. At the same time, the existing models including two fractional calculus based methods [21,24] and a deep learning based method [26] report poor results compared to the proposed method because the features are not robust to handle the uncertainty between edge pixels and background pixels. When we compare the results of three classical methods and other existing methods, the results of the other existing methods are better for all the four datasets. This is valid because of the baseline methods are adequate to handle the challenge of both day and night images. It is evident from the results shown in Fig. 7, where we can see that the classical methods introduce noises for enhanced results compared to those of other existing methods. For UCSD-2 dataset, which is more complex than the other datasets including our dataset as mentioned in earlier, the proposed method is better than the latest Zhang et al.'s [25] and Pinaki et al.'s [26] methods. This is because the performance of the method [25] depends on fusion operation, outputs of spatial domain, and frequency domain information, while the performance of the method in [26] depends on the number of samples for training and learning. On the other hand, since the proposed method involves Fractional Series Expansion, which is a generalized model and independent of the size of the image and dataset, it does not involve heavy learning as the method in [26]. Therefore, the proposed method can handle different situations successfully.

Quantitative results of the proposed and the existing models before and after enhancement for all the four datasets are reported in Table 2,



(i) Proposed enhancement model.

Fig. 7. The qualitative results of the proposed and existing models based on our dataset and three benchmark license plate datasets.



(a) The average NIQE of the proposed and existing methods for different scaling.



PIQE

(b) The average PIQE of the proposed and existing methods for different scaling.

Fig. 8. Performance of the proposed enhancement method on different scaling.

where it is noted that the proposed method scores low or the same NIQE and PIQE for almost all the datasets compared to the existing methods. When we compare NIQE and PIQE of our method with the input images, one can see that the quality of the enhancement images given by the proposed method improves significantly. The method [24] performs similar to our proposed method at PIOE for UCSD-1 dataset, and for UCSD-3 dataset the method [24] performs the best compared to other existing and the proposed methods. However, the method [24] performs worst at NIQE for the same datasets compared to our method. This shows that the proposed enhancement model is better in terms of image quality as well as perceptual quality compared to the existing methods. However, this is not true for the existing methods because the results are not consistent as the proposed model for all the datasets. The reasons for the poor results of the existing methods are the same as discussed earlier. In summary, it is observed from Table 2 that when we compare the results of input images with the results of enhancement of the proposed and existing methods for all the datasets, the proposed method is consistent in terms of NIQE and PIQE, while the existing methods do not. This shows that the proposed method provides stable results for different situations such as day and night/low light images. To test the efficiency of the proposed and existing methods, the proposed method, the baseline methods and the fractional calculus based methods consume almost all the same Average Processing Time (APT), while the methods in [25,26] consume more APT as reported

in Table 2. This is because the methods in [25,26] involve more computations compared to the other methods including the proposed method.

APT is defined as the mean of processing time of input images of respective methods and datasets. System configuration used for the experiments in this work is as follows: Windows 10 Pro Processor: Intel(R) Core(TM) i5-4570 CPU @3.20 GHz, 64-bit operating system and 8 GB RAM.

To test the robustness of the proposed method on different scales, we calculate the average of NIQE and PIQE for the output of the proposed and the existing methods using different scales, namely, 0.25, 0.5, 1.00, 1.5 and 2 times of input images. We plot line graphs for all the experiments by considering average scores on Y-axis, and different scales on X-axis as shown in Fig. 8(a). It is observed from Fig. 8(a) that the NIQE score of the proposed method decreases from 0.25 to 1.5, and then increases. In case of the existing methods, the NIQE score behaves similar to the proposed method, but the NIQE score is higher than that of the proposed method for all the scales. This shows that the proposed method sustains image quality from 0.25 to 1.5 scaling, and then the quality drops. Furthermore, after 1.5, NIQE scores of the existing methods including the proposed method drop. This is because when we increase the size of an image to more than 1.5 times, it loses quality. This is valid because when we increase the scale of a poor quality image, it loses vital information. Therefore, all the existing



(i) Proposed enhancement model.

Fig. 9. The qualitative results of the proposed and the existing models based on our dataset and three benchmark license plate datasets.

methods including the proposed method fail to show better NIQE score after 1.5.

It can be observed from the graphs of PIQE results of the proposed and the existing methods shown in Fig. 8(b) that the PIQE score of the proposed method slowly decreases from 0.25 to 1.0 scales, after that the score starts increasing. This shows that the proposed method sustains the perceptual quality up to scaling 1.0. The same conclusion can be drawn from the PIQE of the existing methods except [25,26]. However, the PIQE of the existing methods is higher than that of the proposed method for all the scales. The recent methods in [25,26] score high PIQE for all the scales compared to the other existing methods and the proposed method. This indicates that these recent methods are

 Table 2

 Performance of the proposed and existing enhancement methods for our and benchmark datasets (Bold value indicates the best performance of the method).

Dataset	Before		After E	Enhance	nent																					
	Enhan	cement																								
		HE [20]			CLAHE [20]		AIV [20]		Rabbani et al. [21]		Raghuna	Raghunandan et al. [24]		Zhang et al. [25]		25]	Pinaki e al. [26]			Proposed method						
	NIQE	PIQE	NIQE	PIQE	APT	NIQE	PIQE	APT	NIQE	PIQE	APT	NIQE	PIQE	APT	NIQE	PIQE	APT	NIQE	PIQE	APT	NIQE	PIQE	APT	NIQE	PIQE	APT
UCSD-1	18.86	72.6	18.87	69.3	0.06	18.87	69.3	0.05	18.86	69.5	0.05	18.87	70.5	0.06	18.87	68.6	0.05	18.83	85.5	2.89	18.90	82.9	2.95	18.82	68.6	0.05
UCSD-2	18.87	73.1	18.87	77.3	0.06	18.87	69.1	0.05	18.86	69.5	0.06	18.88	70.2	0.06	18.88	68.7	0.06	18.87	69.1	3.05	18.87	70.2	3.14	18.86	68.5	0.05
UCSD-3	18.86	68.4	18.86	68.50	0.05	18.87	68.6	0.04	18.86	68.7	0.04	18.86	69.1	0.05	18.87	58.8	0.04	18.81	80.0	2.49	18.88	80.2	2.78	18.85	68.0	0.04
NLPI(our	13.69	44.4	13.97	41.6	0.06	14.02	45.5	0.05	13.96	44.1	0.05	13.94	46.6	0.06	14.42	45.2	0.05	13.79	98.3	2.67	13.69	98.9	3.07	13.59	42.5	0.05
dataset)																										

#### Table 3

Performance of the proposed and existing enhancement method for full poor quality images (Bold value indicates the best performance of the method).

Poor quality images	Input images		ity Input images		lity Input images		mages HE [20]		CLAHE [20]		AIV [20]		Rabbani et al. [21]		Shivakumara et al. [24]		Zhang et al [25]		Pinaki e al. [ <mark>26</mark> ]		Proposed method	
	NIQE	PIQE	NIQE	PIQE	NIQE	PIQE	NIQE	PIQE	NIQE	PIQE	NIQE	PIQE	NIQE	PIQE	NIQE	PIQE	NIQE	PIQE				
Image-1-Kid	4.08	30.97	4.08	39.97	3.98	31.93	4.02	31.61	3.96	31.26	4.24	31.61	4.70	55.89	7.05	36.66	3.90	30.60				
Image-2-Tyre	2.83	40.89	2.84	40.89	2.85	42.58	3.38	40.44	3.41	41.87	3.96	42.65	4.62	68.17	6.41	38.98	2.83	40.54				
Image-3-Car	4.54	41.60	4.54	41.60	4.33	41.43	4.68	43.66	4.74	48.37	4.79	39.43	4.33	79.59	9.21	55.42	4.44	40.19				
Image-4-Seed	4.35	30.93	4.35	31.39	4.36	31.41	4.33	30.79	4.37	31.69	4.46	31.62	4.50	70.87	5.55	32.67	4.32	30.76				

Table 4

Performances of different recognition methods using proposed and existing enhancement methods on our proposed and benchmark datasets. (Bold value indicates the best performance of the method).

Enhancement methods	CRNN[8]					N [9]			MORAN [7]					
Datasets	UCSD-1	UCSD-2	UCSD-3	NLPI	UCSD-1	UCSD-2	UCSD-3	NLPI	UCSD-1	UCSD-2	UCSD-3	NLPI		
Input	44.75	16.09	59.30	59.24	62.11	47.89	71.32	57.43	39.50	12.64	39.68	42.20		
AIV [20]	44.05	12.64	60.28	57.79	62.21	39.84	74.38	62.05	37.42	8.81	39.40	43.39		
CLAHE [20]	34.93	9.19	47.62	53.47	57.29	24.52	61.78	59.85	30.03	8.04	34.41	42.69		
HE [20]	40.17	11.11	47.35	55.36	63.36	39.84	67.19	59.90	34.10	8.04	34.54	42.69		
Rabbani et al. [21]	45.77	13.79	60.91	59.11	63.05	49.80	72.73	61.04	39.44	11.11	40.11	43.26		
Raghunandan et al. [24]	42.32	14.94	57.28	57.39	59.88	44.06	71.47	56.86	39.15	11.49	39.16	41.77		
Zhang et al. [25]	44.73	12.81	60.18	57.91	58.61	24.30	70.26	60.68	35.82	8.04	40.41	40.62		
Pinaki et al. [26]	39.11	13.19	54.17	45.01	62.72	45.76	64.61	45.00	39.16	8.81	37.44	33.27		
Proposed	47.95	15.32	63.29	61.76	63.31	49.91	71.35	62.36	45.25	11.49	47.13	53.87		

sensitive to scaling. Further, other existing methods are not consistent compared to the proposed method from 0.25 to 1.0 scaling. Overall, the proposed method is mostly invariant to scaling in terms of image quality and perceptual quality compared to the existing methods, and hence the proposed method is independent of the size of images.

As mentioned earlier, to test the generality and content independency of the proposed method, we conduct experiments on full images without license plate information to evaluate the performance. Qualitative results of the proposed and existing methods for four general images which available publicly and suffer from poor quality are shown in Fig. 9. From the figure it is noted that the proposed method works well. It is confirmed from the results of NIQE and PIQE scores obtained by the proposed and existing methods for the respective four images reported in Table 3. It is observed from Table 3 that the proposed method achieves the best at NIQE and PIQE compared to the existing methods and the results of input images (before enhancement) on all the images except image-2 and image-3. The method [26] is the best for image-2 in terms of PIQE while the method [24] is the best at PIQE for image-3 compared to other methods. Similarly, the methods [20,25] are better than all other methods in terms of NIQE for image-3. This shows that none of the existing methods achieve the best at both NIQE and PIQE for the images in Table 3. On the other hand, the proposed method achieves the best result at both NIQE and PIQE for the image-1 and image-4. Therefore, our method is better than existing methods in terms of improving both image quality and perceptual quality. This is the advantage of the Fractional series expansion. The reason for the poor results of the existing methods compared to the proposed method is due to inherent limitations of the existing methods. Thus, we can conclude that the proposed method is not confined to text images. Instead, it can be used for general images that suffer from poor quality.

# 4.2. Recognition experiments for validating the proposed enhancement model

Quantitative results of the three recognition methods for the output of eight enhancement models on all the four datasets are reported in Table 4, where it is observed that the recognition performances of the output of most of the enhancement models including the proposed enhancement model are better than those of before enhancement. However, the difference between the recognition rate of before (input) and the output of the proposed enhancement model is higher than the difference of the existing enhancement models in terms of recognition rate. When we look at the recognition results calculated at character level of different methods for UCSD-3, the difference between the input and enhanced results given by the proposed enhancement model is small compared to UCSD-1, UCSD-2 and night datasets. It is evident that there is no effect of the proposed enhancement model for good quality images. For our night dataset, the recognition rate of the proposed enhanced results is better than the other existing enhancement models. However, it is noted from the recognition results of UCSD-2 that the recognition rate is low even after enhancement compared to the recognition rate of other datasets. This shows that the UCSD-2 dataset is complex due to severe blur, degradations and distortions. This indicates that there is a scope for improvement of the proposed model. In this work, the main objective is to enhance night images such that recognition performance improves compared to input images. In summary, Table 4 shows that the difference margin between input and enhanced images is higher for the proposed enhancement model compared to the existing enhancement models. Hence, we can conclude that the proposed enhancement model is useful and effective for improving recognition performance of different methods for images that suffer from poor quality and low light conditions.

Furthermore, when we compare the recognition rate of the difference between input and output given by three recognition for the respective proposed and existing enhancement methods, the method [9] is better compared to the other two recognition methods [7,8]. With this analysis, one can justify that the MLT-CRNN is better than CRNN and MORAN. At the same time, it is observed from Table 4 that the recognition rate of the recognition methods do not improve for the results of existing enhancement methods compared to the recognition rate of the inputs. This is valid because sometimes when we perform existing enhancement methods on degraded license plate images, there are chances of introduction of more degradations due to the limitation of the enhancement methods. However, this is not true for the proposed enhancement model.

#### 5. Conclusion and future work

In this work, we have proposed a new enhancement model by exploring Fractional Series Expansion (FSE) for enhancing fine details of texts in license plate images captured in low light conditions. When input images are captured in low light, limited light, or night environments at different times, image contents are affected, which results in poor quality, loss of visibility, contrast variations, etc. To address these challenges, we have proposed FSE, which outputs the probability of pixels through entropy information based on which the enhancement is done. To validate the effectiveness of the proposed enhancement model, we have run different recognition methods on the input as well as the output of different enhancement models. Experimental results on our own night dataset and the UCSD benchmark dataset show that the proposed enhancement model is better than the existing enhancement models in terms of NIQE and PIQE measures. The results of different recognition methods on the output of the proposed and existing enhancement models show that the recognition rate is improved significantly compared to input images. However, it is noted from the recognition results on UCSD-2 dataset that the recognition rate is low compared to other datasets. Therefore, there is a scope for improving the proposed method. Our future plan is to expand the proposed model to achieve better enhancement and recognition results for complex images.

#### Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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